NUMERICAL ANALYSIS II

MATH 8510

Course Description:

Topics covered in this course include interpolation and approximations, numerical differentiation, numerical integration, and numerical solutions of ordinary and partial differential equations. **3 credits**

Prerequisites:

MATH 1970, MATH 2350, or permission of instructor. Familiarity with computer programming is assumed.

Overview of Content and Purpose of the Course:

This course covers methods for numerically solving ordinary and partial differential equations. Interpolation, numerical differentiation and integration, numerical solutions to ordinary and partial differential equations (parabolic, hyperbolic and elliptic types) are studied. Convergence and stability criterion are developed.

Anticipated Audience/Demand:

Graduate majors in Mathematics, Engineering, or Computer Science needing advanced numerical methods for solving differential equations.

Major Topics:

Interpolation and Approximations:

1) Lagrange Interpolation and Newton Polynomials
2) Error Analysis of Polynomial Interpolation
3) Hermite Interpolation
4) Piecewise-Polynomial Interpolation
5) Cubic Spline Interpolation

Numerical Differential and Integration:

1) Numerical Differentiation
  a. Approximating the Derivative
  b. Richardson Extrapolation

2) Numerical Integration
  a. Introduction to Quadrature
  b. Newton-Cotes Formulas
  c. Composite Rules for Numerical Integration
  d. Adaptive Quadrature
  e. Richardson Extrapolation and Romberg Integration
  f. Gaussian Quadrature

<u>Solution of Differential Equations</u>:

1) Introduction

2) One-Step Methods
- **a.** Euler Method
- **b.** Taylor Series Methods
- **c.** Runge-Kutta Methods
- **d.** Variable-Step Runge-Kutta Methods
- **e.** Higher-Order Differential Equations and Systems of Differential Equations

3) Linear Multistep Methods
- **a.** Adams-Bashforth Methods
- **b.** Adams-Moulton Methods
- **c.** Predictor-Corrector Methods

4) Shooting Methods for Two-Point Boundary Value Problems
5) Finite Difference Methods for Linear and Nonlinear Boundary Value Problems
6) The Rayleigh-Ritz Method

<u>Numerical Solution of Partial Differential Equations</u>:

1) Parabolic Partial Differential Equations
- **a.** Crank-Nicolson Equations
- **b.** Implicit and Explicit Finite Difference Equations
- **c.** Handling Boundary Conditions
- **d.** Matrix Norms
- **e.** Stability and Convergence of the Finite Difference Equations

2) Hyperbolic Partial Differential Equations
- **a.** Characteristics
- **b.** Finite Difference Equations
- **c.** Pade Difference Approximations

3) Elliptic Partial Differential Equations
- **a.** Finite Difference Equations
- **b.** Curved Boundary Formulas
- **c.** Iterative Methods of Solutions for Matrix Equations

4) An Introduction to the Finite Element Method

<u>Methods</u>:

The course will be presented in a lecture-discussion format.

<u>Student Role</u>:

Students must participate in class and complete the course requirements.

<u>Textbook</u>:

Burden, Richard L., and J. Douglas Faires. *Numerical Analysis, 9th ed*. Boston: Brooks/Cole, 2010.

February 2016